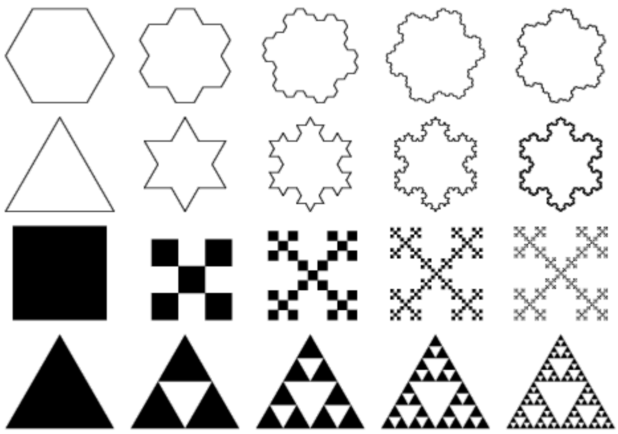
Robogals Science Challenge

Minor Challenge Set #1
STEM Field: Mathematics
Level: Intermediate
Challenge Name: Fun with Fractals
Materials required:

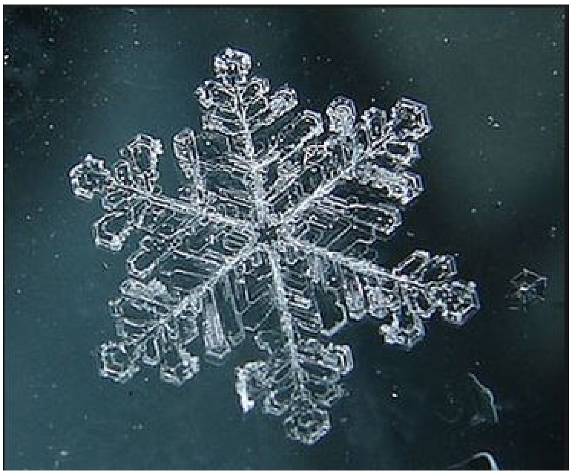
- A4 paper
- Pen
- (Recommended) Access to a printer
- (Optional) Coloured pencils or highlighters
- (Optional) Internet access (laptop/computer)

Introduction:

A fractal is a mathematical shape that exhibits a never-ending pattern. Some examples of mathematical fractals include Koch snowflake, Sierpinski triangles and Mandelbrot fractals. It may sound complex, but all of these fractals are created by the process of iteration: we start with a pattern, then repeat over and over again.



Here is an example of where you can find fractals in nature: the Koch snowflake. We can see that after each iteration, the complexity increases within the shape, which forms the fractal appearance of the snowflake.



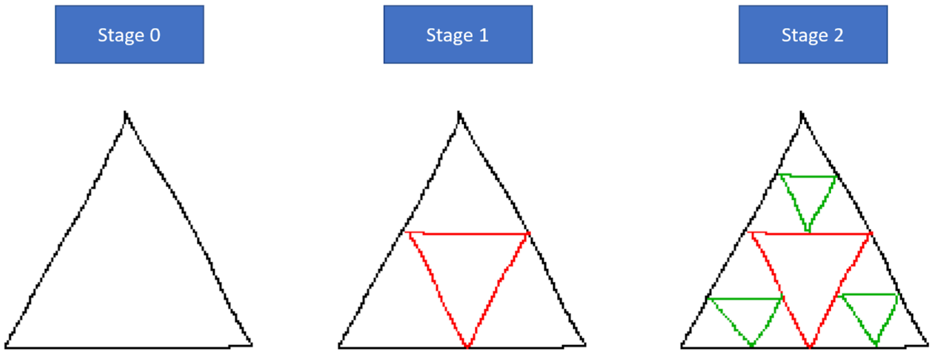
Instructions:

In this project, we will explore Sierpinski triangles, which is named after the Polish mathematician Wacław Sierpiński. The Sierpinski triangle was formally discovered in 1915, however, it has appeared many centuries earlier in artworks. Here is an example of Sierpinski triangles on the floors of churches in Rome.

Our goal is to illustrate how a more complex shape can be formed using simple repetition. This project is split into two parts, so you can complete each part at your own pace.

Part A: Sierpinski's Triangles

1. On an A4 piece of paper, draw an equilateral triangle with sides of 18 cm. We will refer to this as stage 0.
2. Determine midpoints of each side. Use these midpoints as vertices for the new triangle, then draw a new triangle in the centre of the original shape. Colour this new middle triangle using a colour of your choice. This is our stage 1.
3. Repeat step 2 for each remaining triangle. Colour the middle triangles using colours of your choice. This is our stage 2. By the end of this stage, you should have a picture similar to the figure below.
4. Count how many blank / not coloured triangles there are in your picture and fill in the table below.



5. Repeat step 2 and construct your Sierpinski triangle. This is stage 3.
6. By stage 3, how many not coloured triangles are there in your picture? Fill in the table below.
7. Predict how many blank / not coloured triangles you will see in stages 4 and 5. Fill in the table below.
8. Construct your Sierpinski triangles pattern for stages 4 and 5. Count how many blank / not coloured triangles there are, and fill in the appropriate column in the table. Was your prediction correct?

Step	Number of blank / **not** coloured triangles (Predicted)	Number of blank / **not** coloured triangles (Counted)
0	0	0
1	3	3
2		
3		
4		
5		

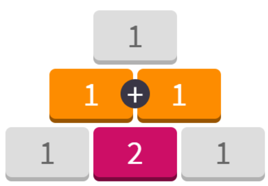
Part B: Pascal's Sierpinski's Triangles

A Pascal triangle is a number pyramid that is created using a simple pattern: it starts with a single "1" at the top, and every following cell is the sum of the two cells directly above. Pascal's triangle was named after Blaise Pascal, a famous French mathematician and philosopher.

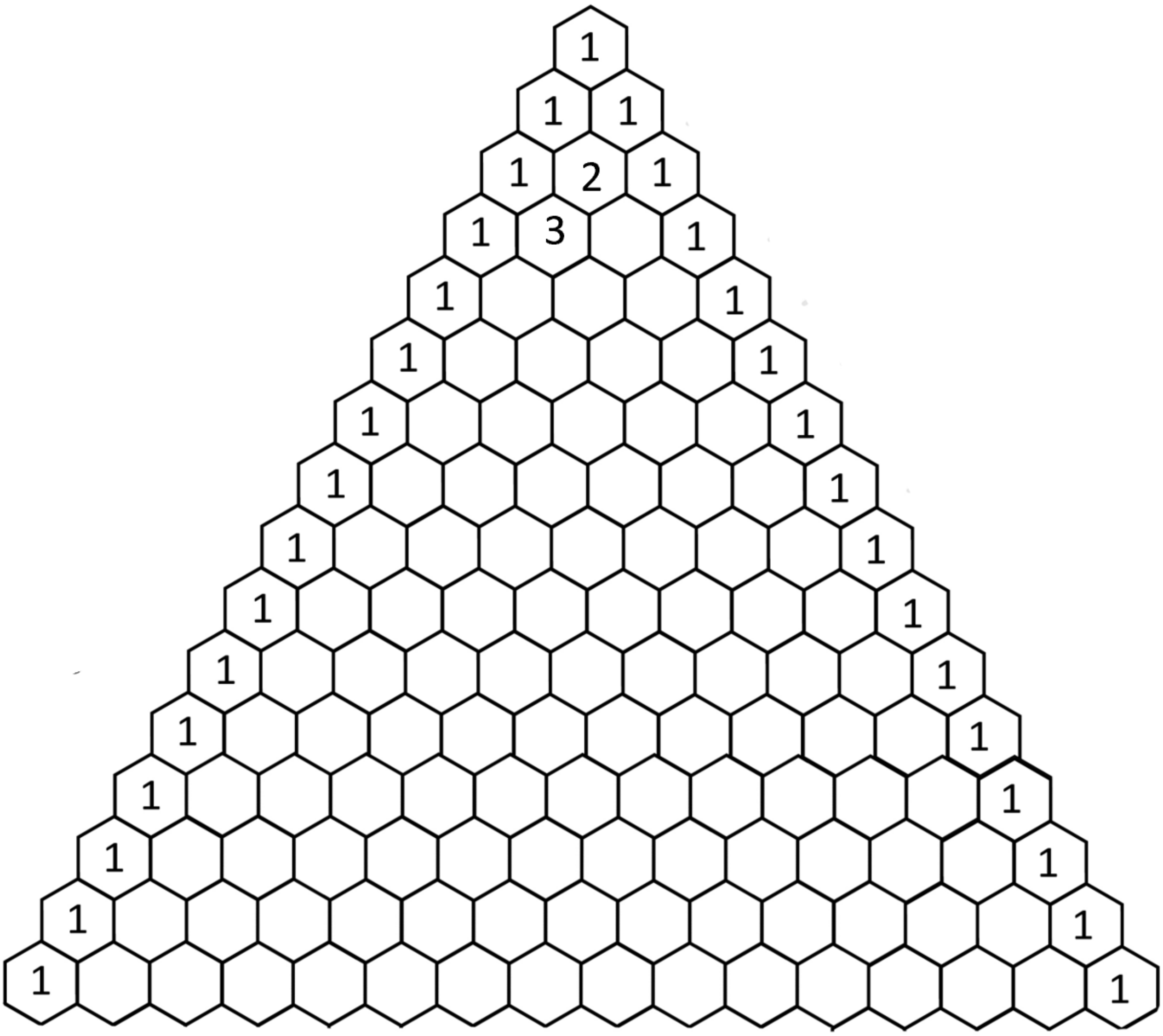
Note: It is recommended that you print out the next page to complete the activity.

Your task is:

1. Complete the Pascal triangle below. Remember that each cell is the sum of the two cells directly above it. For example,



2. After you have completed, colour all odd numbered cells with the same colour.



Extension:

Note: It is recommended that you use a laptop/computer to access the modules below. A browser like Chrome is recommended.

If you are interested in investigating other geometric patterns and more complex mathematical concepts behind fractals, below is an interactive module you can explore after this project.

https://mathigon.org/course/fractals/introduction

Reflection Questions:

- Are there any improvements you would make to this challenge?

- Other than the examples listed in the Introduction section, where else can you find fractals in nature? From your research, list 2-3 examples of natural fractals.
- In Part A, explain your prediction of how many blank / not coloured triangles you will see in stages 4 and 5. How did you arrive at this prediction? Was your prediction correct?
- Can you find a formula to determine the total number of blank / not coloured triangles after any given step? How did you come up with this formula?
- In Part B, after you have coloured all odd-numbered cells, what did you notice? (Hint: The heading of Part B).

Submission Guidelines:

- Submit photos of your results and answers for this project. Include a short summary that addresses the Reflection Questions.

> **Note:** When submitting this Minor Challenge, please upload pictures of your project or experimental setup. Remember, if you want to upload pictures of your Minor Challenge that also include you, please check if it is OK with your mentor first.

- There is a submission form directly on the Minor Challenge page here: https://sciencechallenge.org.au/index.php/minor-challenges/. Fill out the details and make sure you upload your submission.

Learn More! Resources:

- Pascal's Triangles - https://mathigon.org/course/sequences/pascals-triangle

If you enjoyed this challenge, you may want to explore:

- Australian Mathematics Competition - an engaging competition for students from years 3 to 12. More information about the competition can be found here - https://www.amt.edu.au/australian-mathematics-competition.

Sources:

- Peker, Y., n.d. [online] Faculty.randolphcollege.edu. Available at: <http://faculty.randolphcollege.edu/ykurt/Institute2011/Lessons/SierpinskiTriangle.pdf> [Accessed 25 May 2021].

- Weisstein, E., n.d. Fractal -- from Wolfram MathWorld. [online] Mathworld.wolfram.com. Available at: <https://mathworld.wolfram.com/Fractal.html> [Accessed 25 May 2021].